

Flow Dynamics and Mixing of a Transverse Jet in Crossflow— Part II: Oscillating Crossflow

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The present work extends Part I of our study to investigate the flow dynamics and scalar mixing of a turbulent gaseous jet in an oscillating crossflow. Attention is first given to intrinsic flow instabilities under a steady condition. Both power spectral density and proper orthogonal decomposition analyses are applied. For the case with a jet-to-crossflow velocity ratio of 4, the two most dynamic modes, corresponding to jet Strouhal numbers of around 0.1 and 0.7, are identified as being closely linked to the shear-layer vortices near the injector orifice and the vertical movement in the jet wake region, respectively. The effect of oscillation imposed externally in the upstream region of the crossflow is also examined systemically at a jet-to-crossflow velocity ratio of 4. A broad range of forcing frequencies and amplitudes are considered. Results reveal that the dominant structures observed in the case with a steady crossflow are suppressed by the harmonic excitations. Flapping–detaching motions, bearing the forcing frequencies and their subharmonics, become dominant as the forcing amplitude increases. The ensuing flow motions lead to the formation of a long, narrow jet plume and a relatively low mixing zone, which substantially alters the mixing efficiencies as compared to the case with a steady crossflow. [DOI: 10.1115/1.4035809]

1 Introduction

The present work extends Part I of our study [1] on the flow dynamics and scalar mixing of a turbulent gaseous jet in steady crossflow, to investigate the intrinsic flow instabilities and the effect of external excitations in the crossflow.

Over the past three decades, mixing enhancement, especially by means of modulation of jets, has been the subject of considerable research, as summarized in Table 1. Vermeulen et al. [2] used loudspeaker drivers to excite jets in a confined crossflow over a wide range of forcing amplitudes and jet Reynolds numbers. A train of toroidal vortices was generated from the injection orifice, inducing significant increases in jet spread and penetration. An optimum mixing response occurred at a Strouhal number of 0.22, defined as $St = fd/U_j$. Gogineni et al. [3] excited a square jet using four piezoelectric actuators on the conduit sides, causing the low-speed wake region to shrink and the jet penetration to increase. Johari et al. [4] used a solenoid valve to cut off the water jet supply during a portion of the injection cycle. They found that complete modulation of the jet flow altered the jet structure, penetration depth, dilution, and mixing to varying degrees, depending on the pulsing duration and frequency. Eroglu and Breidenthal [5] generated square waves using a solenoid valve and concluded that optimum vortex loop spacing and strength could be achieved in the near field, to increase jet penetration and mixing. M'Closkey et al. [6] quantified the dynamics of actuation of a temporally forced round jet and developed a methodology to set a jet compensation system for open-loop jet control. They observed that the optimal jet penetration and spread occurred with square-wave excitation at subharmonics of the natural vortex shedding frequency of the jet. Narayanan et al. [7] adopted open-loop control forcing at a jet Strouhal number of 0.1 and found this control method to be effective in organizing unsteadiness and enhancing mixing. Shapiro et al. [8] determined the optimal temporal pulse widths for square-wave excitation for a variety of excitation frequencies. Denev et al. [9] introduced swirl to the transverse jets

and found that, although the mixing was intensified near the jet exit as the turbulent kinetic energy and the vorticity of the average flow field increased, the entrainment of the crossflow fluid was attenuated along the counter-rotating vortex pair (CVP) as the jet approached the crossflow tunnel wall and enlarged the wall blocking effect. The overall mixing efficiency, however, remained unchanged. More studies on actively controlled jets in the JICF context are summarized in a recent review by Karagozian [10].

In many engineering applications, including gas turbine engines, fuel injectors are operated at choked conditions to maintain a desired mass flow rate, while disturbances generated in the downstream region (for instance, in the combustor section) may travel upstream and influence the fuel–air mixing process. Therefore, transverse jet evolution and dispersion happen in an intrinsically unsteady crossflow environment. Little has been published, however, in this area. Lam and Xia [11] and Xia and Lam [12] used both laboratory experiments and numerical analyses to investigate the dispersion of a round jet issuing into an unsteady crossflow. They found that the jet effluent was organized into successive large-scale effluent clouds, and the jet width increases at an incremental rate that is strongly affected by the crossflow unsteadiness. Kremer et al. [13] presented a numerical study of a steady round jet issuing into an oscillating crossflow with a sinusoidal velocity profile, based on calculations using the FLUENT 6.2 commercial code. They found that both the velocity and scalar concentration trajectories of the jet plume were slightly dependent on the oscillation amplitudes, but were closely linked to the crossflow oscillation frequencies until the deviation maximum was achieved, at which point the velocity and scalar concentration trajectories began to revert back to the steady profiles. For these studies, water was used as the working fluid for both jet and crossflow.

The present work considers the effects of external excitations in the crossflow in a gaseous turbulent JICF environment. The computational domain is the same as that in Part I of the study. Figure 1 shows a schematic of the boundary configurations specially designed for cases with an oscillating crossflow, to be elaborated in Sec. 3. The paper is organized as follows. Section 2 reviews the existing studies on jet stability and explores the intrinsic flow instabilities in the baseline case with a steady crossflow. Both power spectral density (PSD) and proper orthogonal

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Contributed by the Turbomachinery Committee of ASME for publication in the JOURNAL OF ENGINEERING FOR GAS TURBINES AND POWER. Manuscript received June 14, 2016; final manuscript received December 20, 2016; published online March 21, 2017. Assoc. Editor: Riccardo Da Soghe.

decomposition (POD) analyses are applied to extract the dominant frequencies and structures in the pressure and scalar fields. The findings in this section establish the rationale for specification of the excitation parameters. Section 3 describes the computational setup for the oscillating cases and discusses results for multiple combinations of forcing frequency and amplitude, in a consideration of a broad range of fluctuations under conditions relevant to gas turbine engines. Summary and conclusions are presented in Sec. 4.

2 Stability Analysis of a Jet in Crossflow

Flow and mixing properties in the JICF are dominated by a set of complex, interrelated vortex systems [14], and accurate descriptions of the fundamental dynamics are essential for understanding flow response to external perturbations. While Part I of this study identified the coherent structures and their roles in the mixing process, the present work further explores the intrinsic instabilities of the flow field. Only the results for the case with a jet-to-crossflow velocity ratio, r , of 4 are presented here. All calculations are based on the intermediate grid B with a total of 8.9×10^6 mesh points, as described in Part I of the study. Air at ambient conditions, 1 atm and 300 K, is selected as the working fluid for both the jet and the crossflow. The jet-to-crossflow density ratio is unity in this study; otherwise, the jet-to-crossflow momentum flux ratio, $J = (\rho_j U_j^2)/(\rho_c U_c^2)$, should be considered to account for the momentum effects on the jet evolution. The mass fraction of the jet fluid is set as the scalar under investigation. The bulk velocity of the crossflow is 40.0 m/s. The jet is assumed to have a fully developed velocity profile with a center-line velocity of 160.0 m/s at the inlet and enters the crossflow through a wall orifice of diameter 1.27 mm, as specified in Part I. The time steps in all the calculations are fixed at 2.0×10^{-8} s, for the purposes of time-accurate data extraction. For direct comparison, snapshots of the temporal evolution of vorticity magnitude and scalar concentration of both the steady and forced cases are shown side by side in Figs. 2 and 3.

Bagheri et al. [15] carried out global stability analysis of a JICF with $r=3$ based on a steady-state solution of the incompressible Navier–Stokes equations. They observed two groups of global modes: low-frequency wake modes associated with the vortical structures in the wake region and shear-layer modes with high frequencies and large amplitudes located at the CVP. Megerian et al. [16] presented an experimental exploration of shear-layer instabilities for $1 \leq r \leq 10$ and $r \rightarrow \infty$ at jet Reynolds numbers of 2000 and 3000. They reported that the JICF transitioned from a

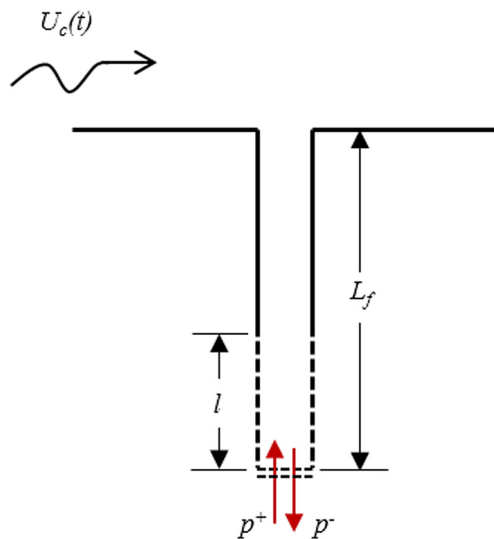


Fig. 1 Schematic of the boundary configurations

Table 1 Studies of modulated single jet in crossflow

Ref.	Jet conditions				Modulations				
	Fluid	r	d (mm)	U_j (m/s)	Re^a	U_e/U_j	f (Hz)	St^b	Approach
Vermeulen et al. [2]	Air	1.3–4.6	19.93	10.7–42.7	11,124–43,518	0–1.67	208	0.097–0.389	Loudspeaker
Gogineni et al. [3]	Air	1.0	15.2 square	NA	618	NA	5	NA	Piezoelectric actuator
Johari et al. [4]	Water	5, 10	3.0	0.75, 1.5	2250, 4500	NA	0.5–5	0.0009–0.025	Square wave, solenoid valve
Eroglu and Breidenthal [5]	Water	2.3, 4.4	NA	NA	650, 6200	NA	0–20	0–0.71, 0–0.11	Square wave, solenoid valve
M'Closkey et al. [6]	Air	2.58	7.5	3.1	1500	1.7/3.1	73.5, 110	NA	Sine and square waves, loudspeaker
Narayanan et al. [7]	Air	6	3	24.4	5000	0.3, 0.25	680, 1000, 1300, 1500	0.085, 0.125, 0.163, 0.190	Sine wave, valve actuator
Shapiro et al. [8]	Air	2.58, 4.0	7.62	2.8, 4.3, 5.6, 7.2	1420, 2200, 2830, 3660	1.5/2.8, 1.8/4.3, 2.2/5.6, 2.3/7.2	220, 454, 600, 870	0.60, 0.80, 0.82, 0.92	Square wave, loudspeaker

^a $Re = U_d d / \nu$, where U_d and d are jet velocity and diameter, respectively.

^b $St = fd / U_j$, where f is the forcing frequency.

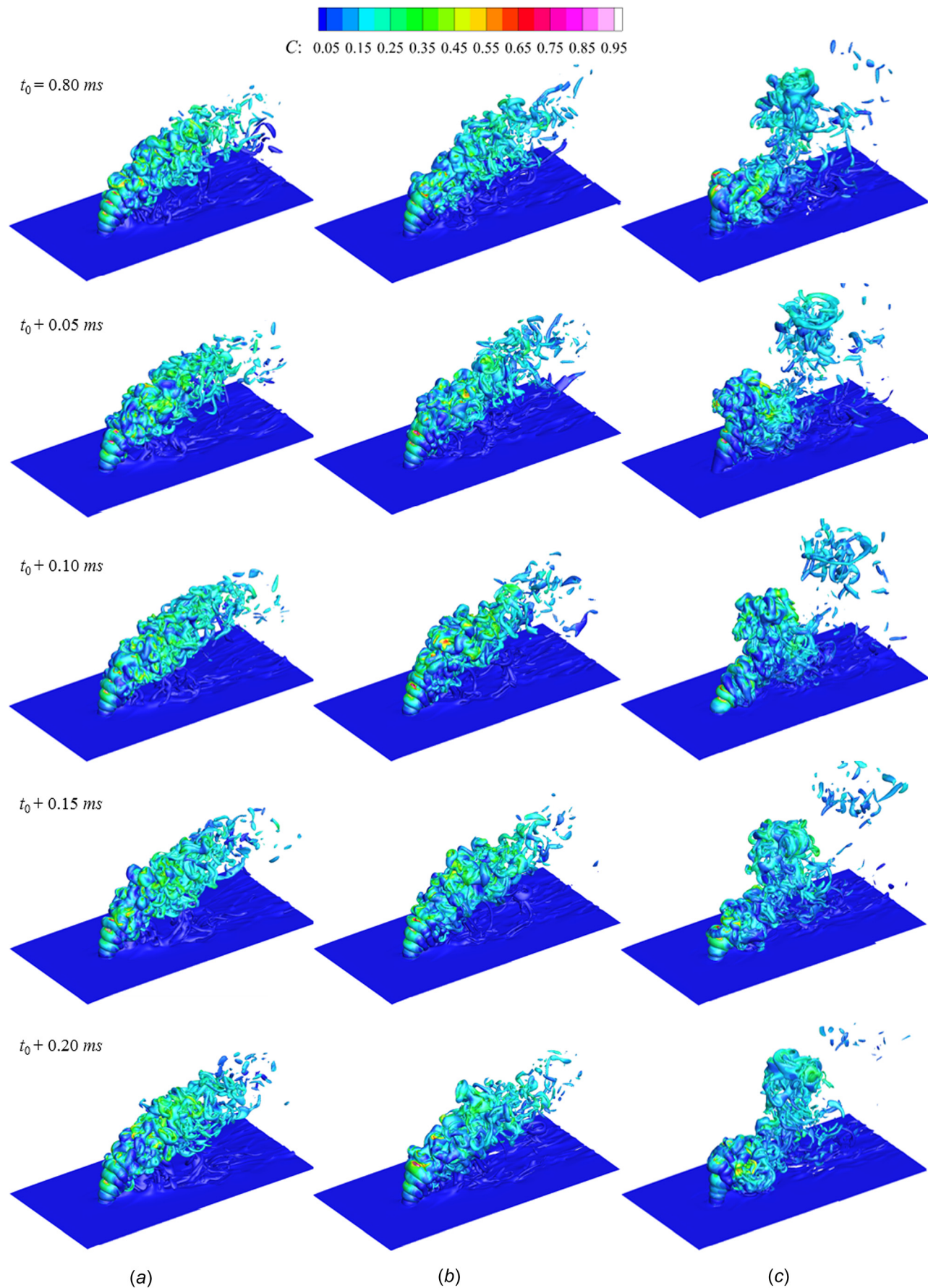


Fig. 2 Temporal evolution of isosurface of vorticity magnitude $|\Omega| = 1.0 \times 10^5/\text{s}$ colored by scalar concentration: (a) steady crossflow, (b) case II: 5 kHz, 10%, and (c) case II-2: 5 kHz, 50%

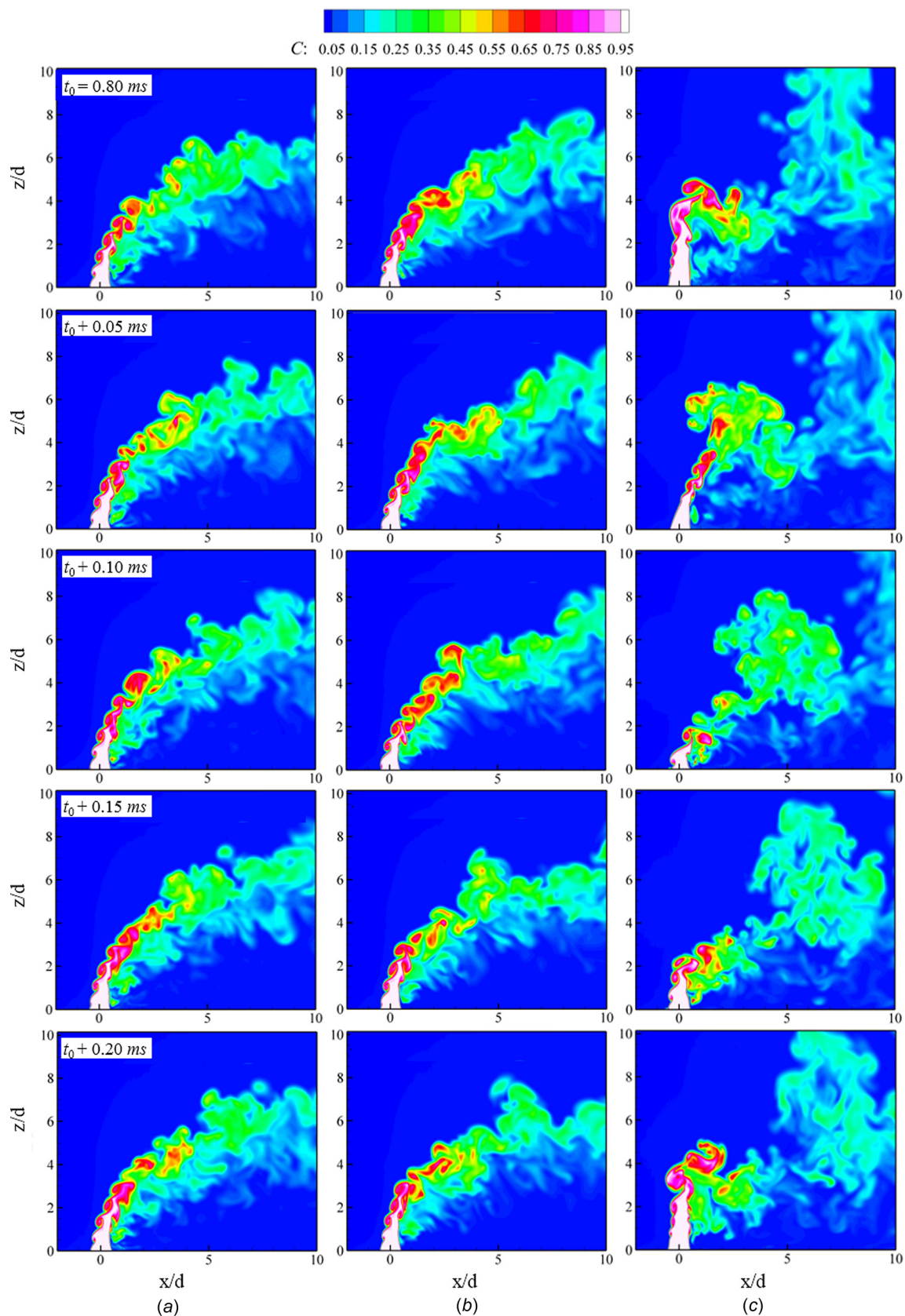


Fig. 3 Temporal evolution of scalar concentration in the jet-center plane: (a) steady crossflow, (b) case II: 5 kHz, 10%, and (c) case II-2: 5 kHz, 50%

convectively unstable flow at a high r to a globally or absolutely unstable flow at lower r . They also suggested that the JICF systems were controlled by the forcing strategies: For $r > 3.5$, relatively low amplitude excitations can enhance convective instability and promote mixing [7]; for $r < 4.0$, the jet can be self-excited, and thus, strong forcing, especially at a distinct, externally imposed time scale, is required to adjust jet penetration and spread [6].

In the present work, both PSD and POD analyses are conducted to capture the dominant dynamic structures and their corresponding frequencies for the $r = 4.0$ case.

2.1 Power Spectral Density Analysis. The instantaneous pressure and scalar concentration are recorded at probes 07, 20, and 34, as shown in Part I, after the flow field reaches its statistically steady state. These probes are located in the jet potential core, the near field, and the far field of the jet plume, respectively. Data are recorded at each time step during a period of 2.5 ms corresponding to $80d/U_c$. This sampling process ensures that signals with frequencies between 0.4 and 10^4 kHz are captured. Figure 4 shows the frequency spectra determined by the fast Fourier transformation (FFT) technique. Note that the very low-frequency spikes visible in the pressure field are the result of the finite sampling time and do not represent internal flow dynamics. Because of the jet's large velocity and small diameter, the characteristic frequencies are high, on the order of 10 kHz. Two types of peaks are observed in general: one at a distinct frequency around 14.5 kHz, corresponding to a jet Strouhal number of 0.12, and the other distributed in the range of 80–100 kHz. The first peak is seen only in the pressure field and is seen both in the jet core at probe 7 and in the far field at probe 34. The second peak is present in both the pressure and scalar concentration fields and is observed only in jet potential core at probe 7; it disappears in the downstream probes.

2.2 Proper Orthogonal Decomposition Analysis. Proper orthogonal decomposition (POD) analysis is an effective technique for extracting useful information from large amounts of data in experimental and numerical studies. It is particularly useful in dealing with complex turbulent flows dominated by energetic coherent structures; POD finds a set of ordered orthogonal basis functions, $\phi_j(\mathbf{x})$, $j = 1, 2, \dots$, for a given flow property $f(\mathbf{x}, t)$, so that samples of $f(\mathbf{x}, t)$ can be expressed optimally by the first n basis functions. The projection of $f(\mathbf{x}, t)$ onto the first n basis

$$\hat{f}(\mathbf{x}, t) = \bar{f}(\mathbf{x}) + \sum_{j=1}^n a_j(t) \phi_j(\mathbf{x}) \quad (1)$$

has the smallest error, defined as $E(\|f - \hat{f}\|^2)$. Here, $a_j(t)$ is the temporal variation of the j th mode; $E(\cdot)$ and $\|\cdot\|$ represent the time average and a norm in the L^2 space, respectively. A more comprehensive discussion of this methodology is available in Ref. [17].

Application of POD in the JICF context has been demonstrated in several works. In Guan's parametric numerical studies [18], the energy contents and evolution processes of the selected bases were obtained. Meyer et al. [19] applied POD to their experimental results and reported that wake vortices are the dominant dynamic structures in an $r = 3.3$ case, and shear-layer vortices are dominant in an $r = 1.3$ case. In the present work, POD analyses are performed at two slices, $y = 0$ and $x/d = 10$, covering the near and far fields. The entire procedure has been validated by flow field reconstruction (not shown here). Results for cases with and without external excitations are presented together in Figs. 5–7 for direct comparison. The steady crossflow case is discussed here, and the oscillating crossflow cases will be explained in Sec. 3.

Figure 5 shows the POD results for the fluctuating pressure in the jet-center plane. Energy levels of the most energetic modes

are plotted in a descending order. Also shown are the FFT of the first five modes and mode shapes of the first three modes. Since the POD analysis is sensitive to data selection, the focus region is confined to $-2 \leq x/d \leq 6$ and $-2 \leq z/d \leq 8$ (region with $z/d < 0$ is not shown here). This area covers the early jet development regions where the jet–crossflow interactions are most intense. In Fig. 5(a) for the steady crossflow case, the first two modes have the same structures of shear-layer vortices but with a 180 deg phase difference with respect to each other. The phase shift between these modes indicates that alternating vortices are convecting in the direction of the jet trajectory [19]. The combined energy level of the first two modes is over 70%, which suggests that the fluctuating pressure field is dominated by shear-layer instabilities. Both modes have a frequency of $f \approx 92$ kHz and $St \approx 0.73$. The third mode is at $f \approx 13.5$ kHz and $St \approx 0.12$, and has a 10% energy possession. This mode shows high-pressure regions piled up close to the crossflow bottom wall ahead of and under the jet plume, indicating a compressing effect of jet penetration on the crossflow.

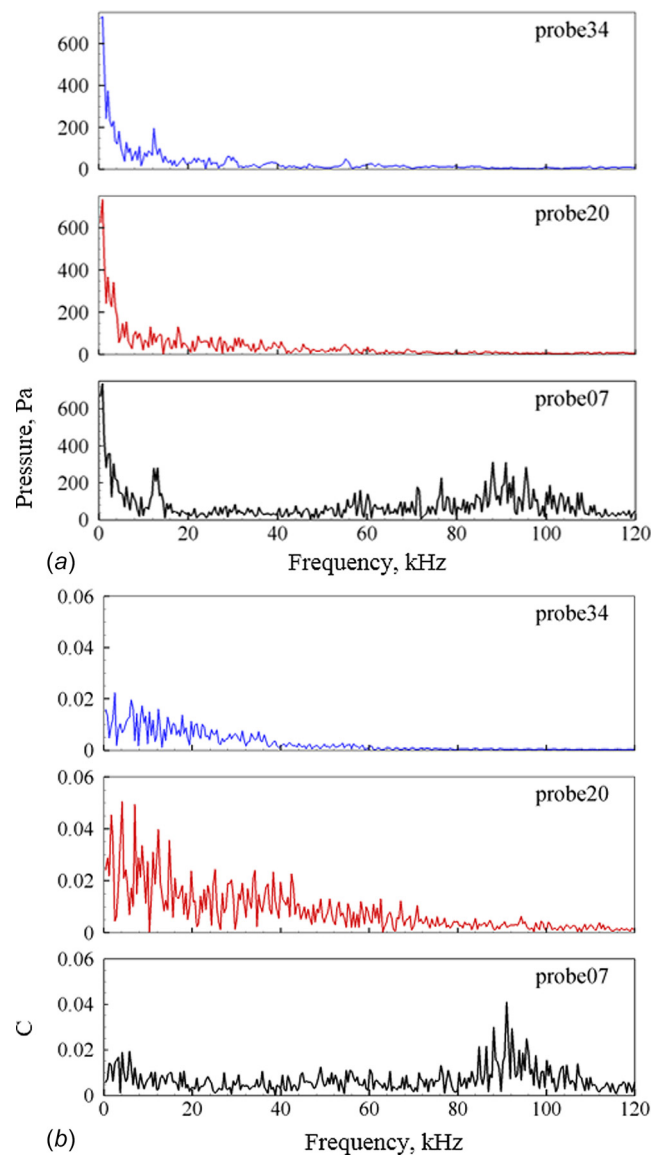


Fig. 4 PSD of instantaneous (a) pressure and (b) scalar concentration at three probe locations for the case with steady crossflow. Note that the very low-frequency spikes visible in the pressure field are the result of the finite sampling time and do not represent internal flow dynamics.

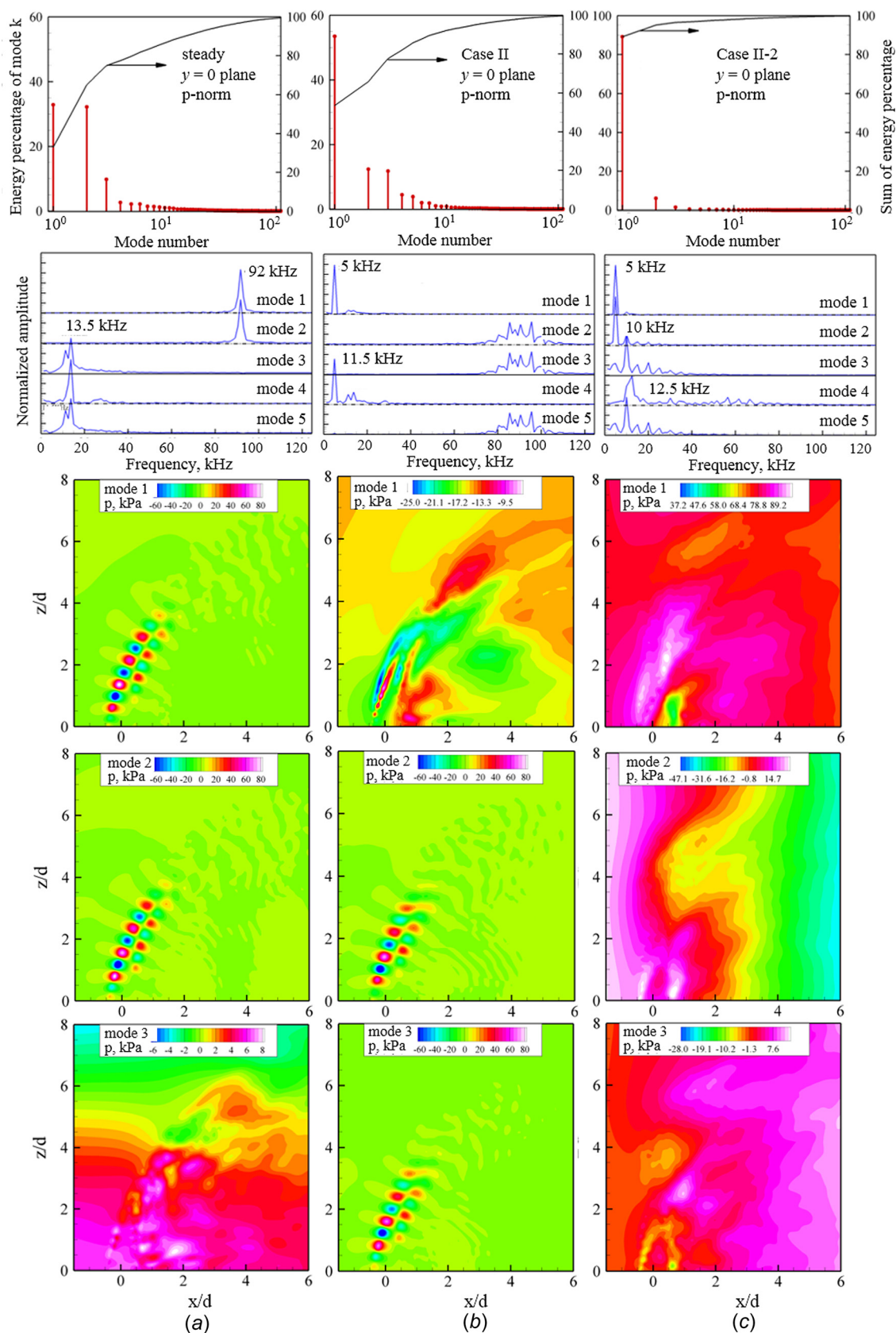


Fig. 5 POD results for the fluctuating pressure in the jet-center plane: (a) steady crossflow, (b) case II: 5 kHz, 10%, and (c) case II-2: 5 kHz, 50%

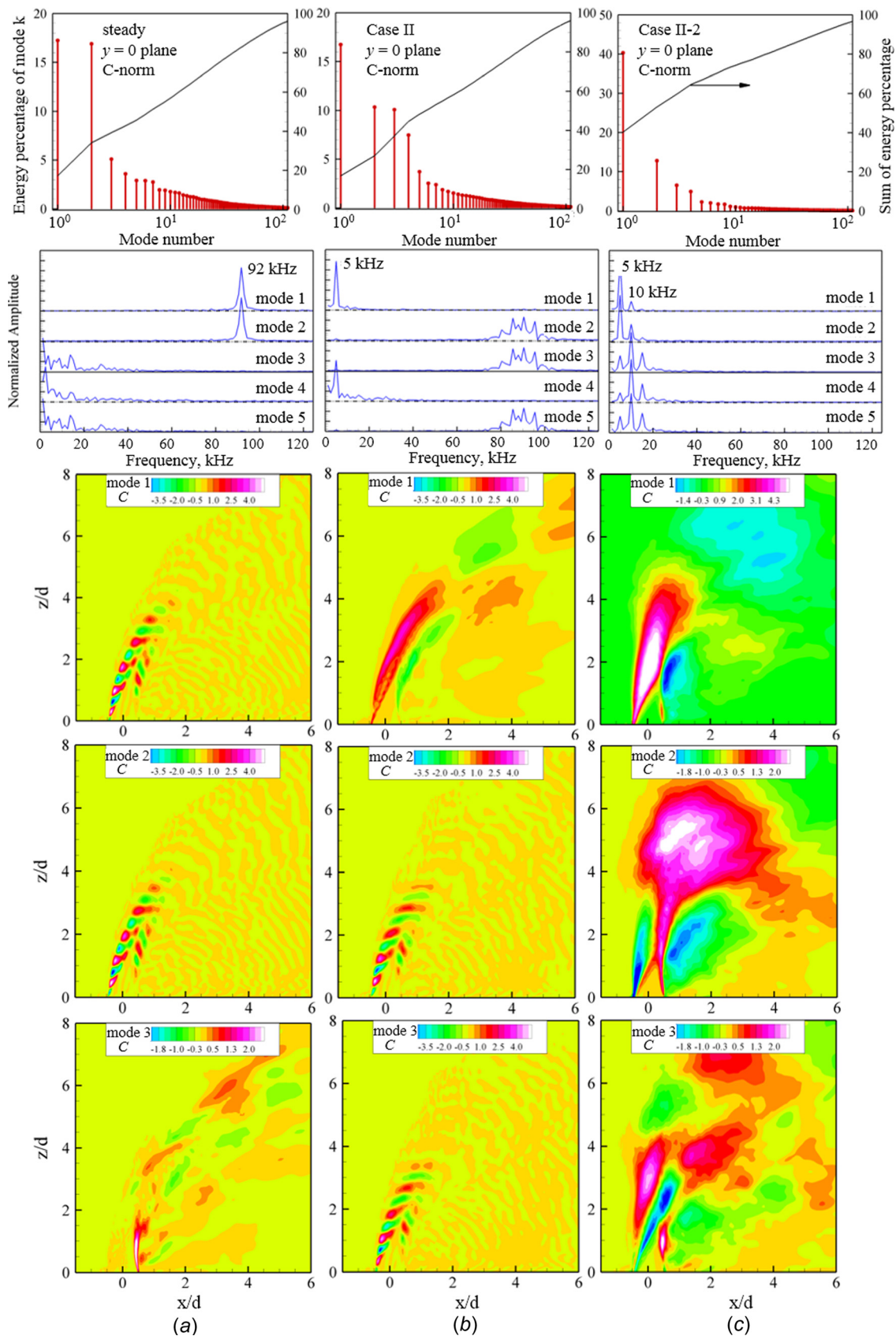


Fig. 6 POD results for the fluctuating scalar concentration in the jet-center plane: (a) steady crossflow, (b) case II: 5 kHz, 10%, and (c) case II-2: 5 kHz, 50%

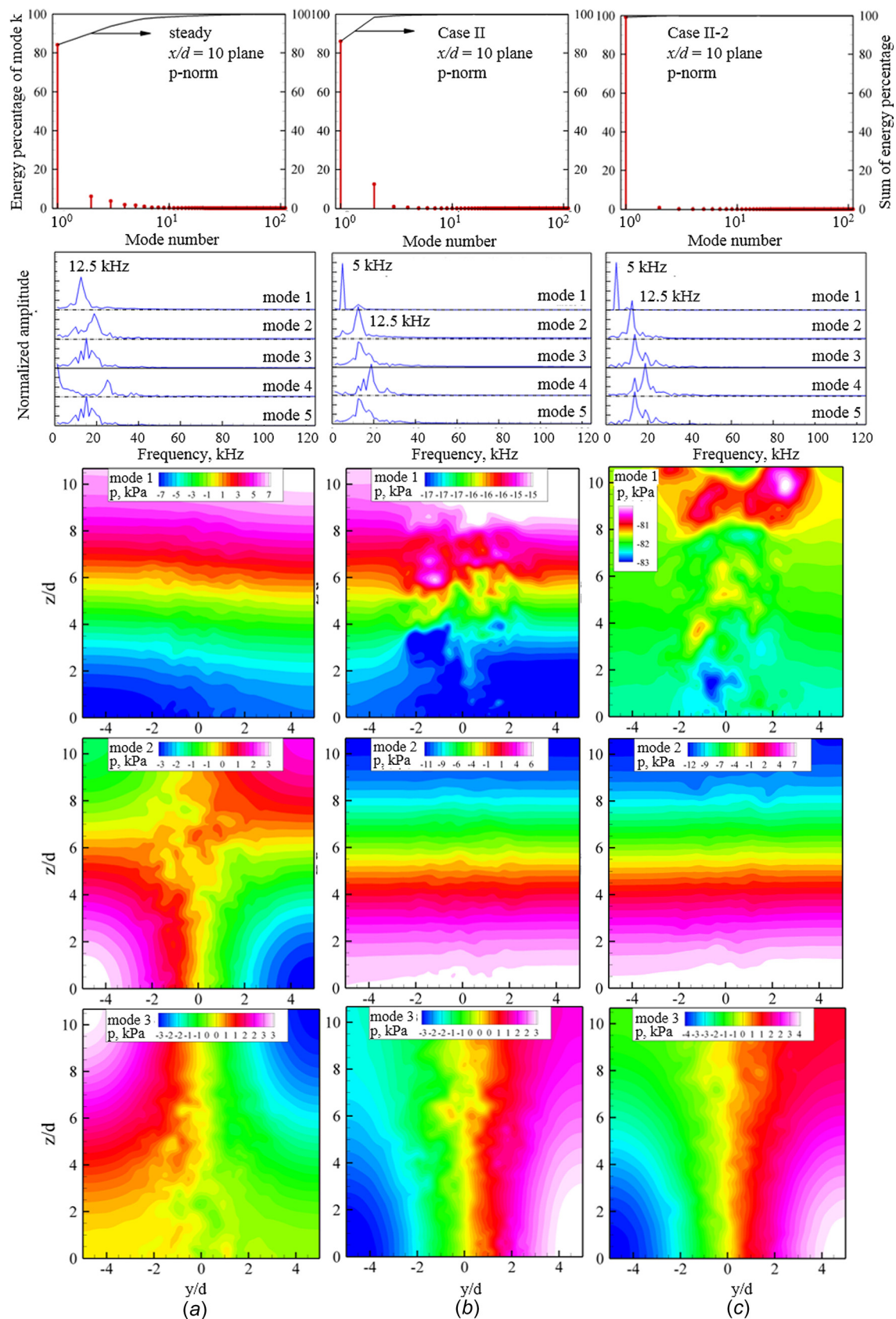


Fig. 7 POD results for the fluctuating pressure in the $x/d = 10$ plane: (a) steady crossflow, (b) case II: 5 kHz, 10%, and (c) case II-2: 5 kHz, 50%

Table 2 Strouhal numbers associated with the jet near-field shear layer

	Re	r	St
Camussi et al. [20]	220	2.2	0.30
Megerian et al. [16]	2000	4.1	0.80
Megerian et al. [16]	3000	4.1	0.95
Rudman [21]	2000	5	0.7–0.75
Narayanan et al. [7]	5000	6	0.10 (broad peak)

The jet shear-layer instabilities tend to be affected by operating conditions, which are typically described by the Strouhal number. Table 2 summarizes a wide range of Strouhal numbers associated with the jet near-field shear layers, documented in the literature. Megerian et al. [16] found that at the same jet Reynolds number, different r leads to different levels of skewing in the velocity profiles near the jet exit, which then alter the shear-layer momentum thicknesses, frequencies, and amplitudes of the unstable modes [22]. From the spectra data of cases with $1 \leq r \leq 10$ and $r \rightarrow \infty$, they observed the early shear-layer mode at $St \approx 0.7$ in the immediate vicinity of the jet exit and the preferred mode for free jets at $St \approx 0.5$ in the downstream region. As the crossflow velocity increased from zero (r decreases from infinity), multiple modes or peaks appeared in the range of $0.5 \leq St \leq 0.7$. For $r \leq 3.5$ (in some cases, $r \leq 4$), the jet exhibited a distinct fundamental mode at the nozzle exit. This dramatic change in the spectral characteristic suggests the presence of a globally unstable mode, which was confirmed at $r = 3.0$ in Ref. [15].

Figure 6 shows the POD results for the fluctuating scalar concentration in the jet-center plane. Compared to Fig. 5(a), the mode distributions in Fig. 6(a) are relatively uniform. The first two modes have shear-layer vortex structures at a frequency of 92 kHz and take the largest share, more than 35%, of the energy. The third mode shows alternating positive and negative values of scalar fluctuations along the jet trajectory and has a high-valued zone on the leeward side of the jet plume. It combines responses of the mixing field to the delayed roll up of the shear layer on the leeward side and to the vorticity breakdown at the end of the near field. The energy levels are more evenly distributed in the scalar field than in the pressure, indicating more gradual scalar mixing.

Figure 7 shows the POD results for the pressure oscillations in the far field at $x/d = 10$. The focus region extends to $-5 \leq y/d \leq 5$ and $0 \leq z/d \leq 11$. In Fig. 7(a), an overwhelmingly dominant mode with an energy share of more than 80% is detected at a frequency of 12.5 kHz, close to that of the third mode at 13.5 kHz shown in Fig. 5(a), and this mode is identified as the compressing effect of the jet penetration on the crossflow. Isobars in the mode shape show transverse movements of the flow field; that is, the jet plume flaps up and down in the far field. Modes 2 and 3 represent the spanwise movements and share only small portions of the energy.

3 Effects of Oscillations in the Crossflow

3.1 Case Description. The schematic in Fig. 1 shows the boundary configurations for a jet injected into an oscillating crossflow. Before the activation of external excitations, a baseline case with a steady crossflow is calculated until its statistically steady state is achieved, which provides the initial conditions for the forcing cases. The round jet orifice is centered at the origin of the domain. The computational domain for the crossflow extends to $-5 \leq x/d \leq 16$, $-5 \leq y/d \leq 5$, and $0 \leq z/d \leq 11$ in the streamwise, spanwise, and transverse directions, respectively. The jet pipe is long enough to ensure that the pipe flow is fully developed before the jet exits the flush-wall orifice into the crossflow. The solution in the jet pipe is updated at each time step, providing instantaneous flow information to the jet–crossflow interaction region.

Excitations are imposed by periodically varying the crossflow velocity U_c at the crossflow duct inlet, such that

Table 3 Case conditions

	f_F (kHz)	St_F	α (%)	p^a/P_0^a (%)
Case I	2	0.06350	10	1.61
Case II	5	0.15875	10	1.61
Case III	10	0.31750	10	1.61
Case II-1	5	0.15875	20	3.22
Case II-2	5	0.15875	50	8.10

Note: $p^a \sim \bar{\rho} \bar{a} u^a$.

^aProvided $\bar{\rho} = 1.2 \text{ kg/m}^3$ and $\bar{a} = 340 \text{ m/s}$.

$$U_c(t) = U_0(1.0 + \alpha \sin(2\pi f_F t)) \quad (2)$$

where U_0 is the mean crossflow velocity, and f_F and α denote the forcing frequency and relative amplitude, respectively. Table 3 summarizes the five excitation conditions that are investigated. The influence of frequency is revealed by a comparison of cases I, II, and III, while the effect of amplitude is assessed between cases II, II-1, and II-2. The forcing frequencies have the same order as the dominant intrinsic frequencies and are in a range observed as relevant in the gas turbine community. At the duct outlet boundary, the zero-gradient condition is maintained throughout the calculations as in the steady crossflow case, where nonreflective characteristics have been confirmed. Artificial wave reflection is further ruled out with a buffer zone downstream. More detailed information is provided in Ref. [23].

The inlet of the jet pipe is modeled as a rigid boundary, where the incident waves are perfectly reflected back to the computational domain. The complex amplitudes of the incident and reflected waves, P^+ and P^- , satisfy that

$$P^+ = P^- \quad (3)$$

$$p^a(x, t) = P^-(e^{ikx} + e^{-ikx})e^{-i(\omega t + M k x)} \quad (4)$$

$$u^a(x, t) = \frac{P^-}{\bar{\rho} \bar{a}}(e^{ikx} - e^{-ikx})e^{-i(\omega t + M k x)} \quad (5)$$

where ω is the wave frequency. p^a and u^a are the acoustic pressure and velocity, respectively. k is the modified wave number

$$k = \frac{\omega/\bar{a}}{1 - \bar{M}^2} \quad (6)$$

$\bar{a}$ and $\bar{M}$ are the sound speed and Mach number of the mean flow, respectively. The collaborative pressure $P_j(x, t)$ and velocity $U_j(x, t)$ are

$$P_j(x, t) = \bar{P}_j(x) + p_j^a(x, t) \quad (7)$$

$$U_j(x, t) = \bar{U}_j(x) + u_j^a(x, t) \quad (8)$$

where $\bar{P}_j(x)$ and $\bar{U}_j(x)$ are the mean flow pressure and velocity; $p_j^a(x, t)$ and $u_j^a(x, t)$ are the acoustic pressure and velocity.

Notice that $p^a \sim \bar{\rho} \bar{a} u^a$, thus

$$\frac{p^a}{\bar{p}} \sim \frac{\bar{\rho} \bar{a} u^a}{\bar{\rho} R T} = \frac{\bar{a} u^a}{\bar{a}^2/\gamma} = \gamma \frac{u^a}{\bar{a}} \quad (9)$$

Since the acoustic velocity is small compared to the sound speed, the acoustic pressure remains small compared to the mean flow pressure, even with a 50% velocity oscillation. The last column in Table 3 shows the relative pressure magnitudes.

3.2 Instantaneous Flow and Scalar Fields. As a rough estimate, given the speed of sound as $\bar{a} = 340 \text{ m/s}$, the wavelengths,

$\lambda = \bar{a}/f$, of the external excitations at frequencies of 2, 5, and 10 kHz are 17.0, 6.8, and 3.4 cm, respectively. The diameter of the jet, 1.27 mm, is much smaller than these wavelengths, and even the computational domain is not long enough to cover an acoustic wavelength. This means that the computational domain is acoustically compact, and acoustic-induced variations are expected to be indiscernible in space. Therefore, visualization of the acoustic effects is only presented based on the temporal evolution of the flow field. The oscillations are activated for 2.5 ms in all forcing cases, corresponding to 5.0, 12.5, and 25.0 forcing cycles for the 2, 5, and 10 kHz cases, respectively.

Figure 2 shows series of isosurface of vorticity magnitude for the steady case and two unsteady cases—cases II and II-2—at the forcing frequency of 5 kHz. These snapshots are recorded at the same time instants and are colored by the scalar concentration. The relative velocity magnitude of oscillation is 10% in case II and 50% in case II-2. The data are recorded 0.08 ms after the excitation is turned on for the latter two cases. The five snapshots are evenly distributed over a time span of 0.2 ms, corresponding to one forcing cycle. Wrinkled surfaces with spanwise vortices in the early jet shear-layer regions and fine structures in the far fields appear in all three cases. In the steady case, the vorticity plume evolves continuously in the downstream direction and shows no apparent time-dependency, whereas in case II, the plume deforms and twists, and a time-evolving upper edge and nonuniform region are created in the far field. The oscillatory phenomena are more pronounced in case II-2. Vorticity appears periodically in the shortened shear-layer region, expands rapidly, breaks down from the jet core, and disperses in the far field. The plume flaps at the excitation frequency, inducing temporal-spatial discontinuity in the vorticity field.

To further visualize the flapping of the jet plume in the oscillating cases, Fig. 3 shows temporal evolution of the scalar concentration in the jet-center plane, with the same time sequence as in Fig. 2. The shear-layer vortices appear in all snapshots, but their size diminishes as the forcing amplitude increases. In case II-2, the length of the jet potential core varies sharply in one forcing cycle; the jet plume resonates with the crossflow and swings vigorously, causing a significant amount of the jet fluid to break from the potential core and disperse into the crossflow. The sizes of the detached jet pockets are larger than those in the steady case, causing more spatial inhomogeneity.

Once the acoustic-induced instabilities in the flow field are visualized, POD analysis is performed to retrieve the underlying mechanism. Figure 5 shows results from POD analysis applied to the fluctuating pressure field in the jet-center plane. Compared to the steady case, the modes of shear-layer vortices—modes 2 and 3 in case II—are suppressed, and their combined energy contribution decreases to 30%. The distinctive peaks of these modes in the frequency domain are smoothed to broad plateaus with relatively lower magnitudes. A new mode—mode 1—appears and takes more than 50% of the energy, revealing its dominance in constituting the fluctuating pressure field. Large pressure variations are observed in the main plume, the wake region, and the end of the jet core. This mode represents the combined effects of the flapping of the jet in the streamwise direction and the bulk breakdown along the jet trajectory. Since the jet resonates with the crossflow, the frequency of this mode complies with the external excitation and has a peak at 5 kHz. The influence of excitation is even clearer in case II-2. The first flapping-detaching mode takes around 90% of energy. The second mode, with about 10% energy share, represents another flapping movement. The modes of the shear-layer vortices phase out in this case, and all the dominant modes are at the external excitation frequency or its subharmonics (for example, 10 kHz in mode 3). It is worth noting that even at a pressure oscillation of 8.1% of ambient pressure, corresponding to a 50% velocity oscillation, flow structures can be substantially changed by the external excitation.

Figure 6 shows the POD results of the fluctuating scalar concentration field in the jet-center plane. As seen in the pressure

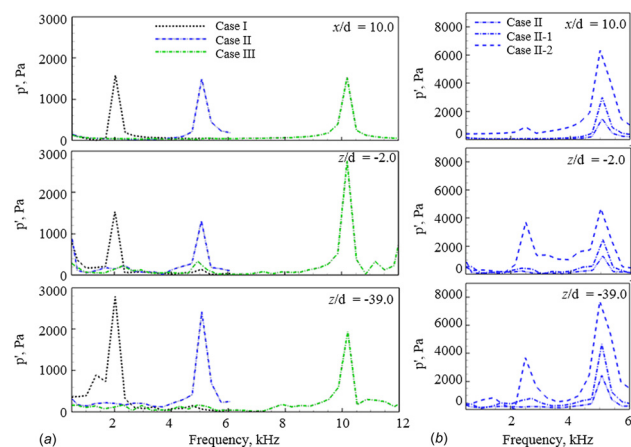


Fig. 8 PSD of fluctuating pressure at three locations: (a) 10% velocity oscillation at three frequencies and (b) three velocity oscillations at 5 kHz frequency

fluctuations, the flapping-detaching mode at the forcing frequency becomes the dominant motion, accounting for more than 15% of the energy in case II. The shear-layer modes degenerate to minor roles and their distinct frequencies are replaced by broad plateaus. These phenomena are more prominent in case II-2, where the first

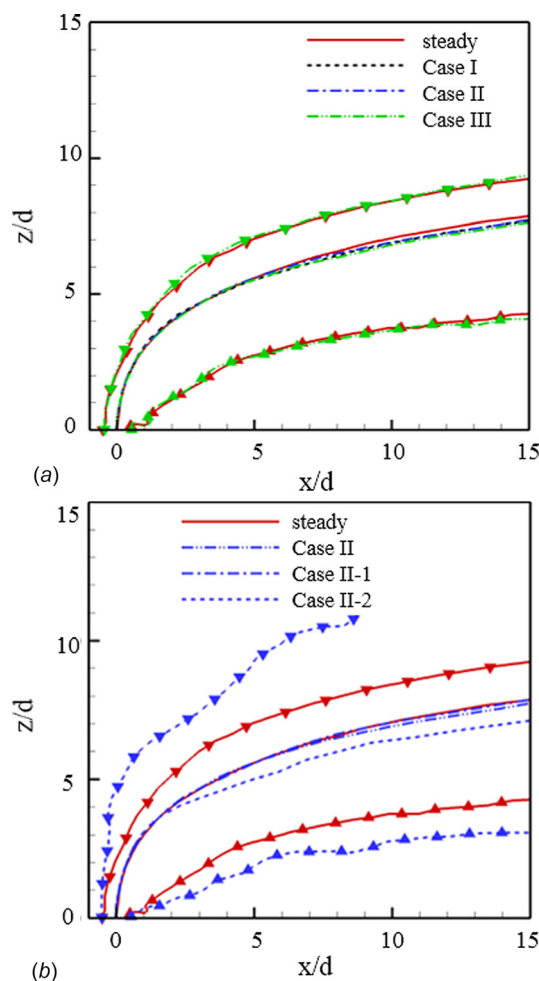


Fig. 9 Time-averaged trajectories of the jet in the jet-center plane: (a) 10% velocity oscillation at three frequencies and (b) three velocity oscillations at 5 kHz frequency

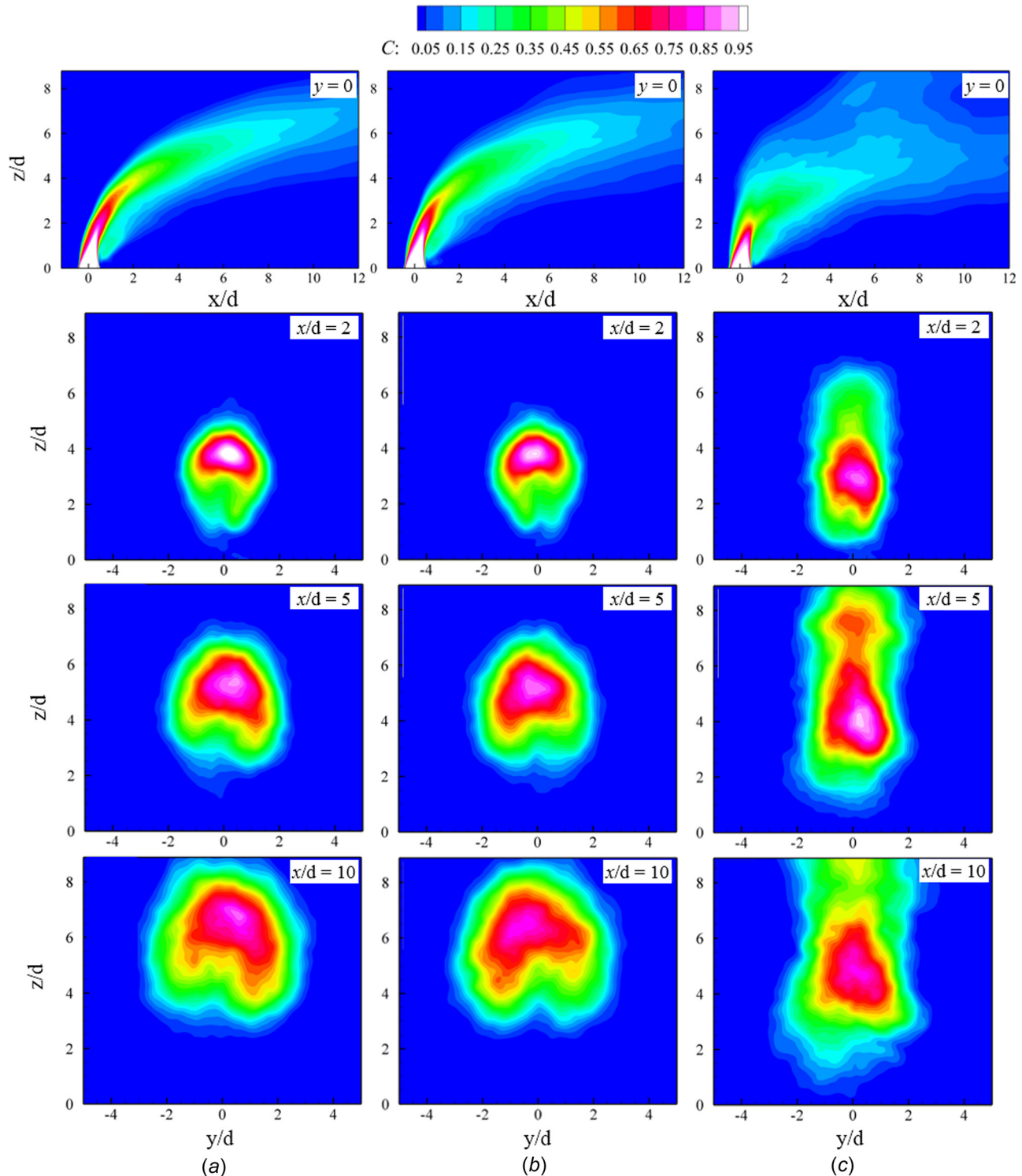


Fig. 10 Time-averaged scalar concentration in the jet-center and transverse planes: (a) steady crossflow, (b) case II: 5 kHz, 10%, and (c) case II-2: 5 kHz, 50%

mode possesses a share of about 40% energy. As the forcing amplitude increases, the regions with negative and positive values of scalar fluctuations enlarge and strengthen. Since the shear-layer vortices play an important role in the early crossflow entrainment, prevalence of the flapping modes in the jet-core region may hinder the engulfing or bulk mixing of the crossflow in the near field.

Figure 7 shows the POD results for the fluctuating pressure in the transverse plane at $x/d = 10$. The modes representing the

transverse and spanwise movements appear as the second and third most dominant structures for both cases II and II-2, while the first mode with a frequency of 5 kHz contains almost all the energy and its mode shape differs from any of the modes observed in the steady case. Two high-pressure zones are found above the jet plume region, while in the scalar analysis (not shown here), two high scalar zones appear in the jet plume and resemble the CVP. In general, the phenomena observed in the POD analyses

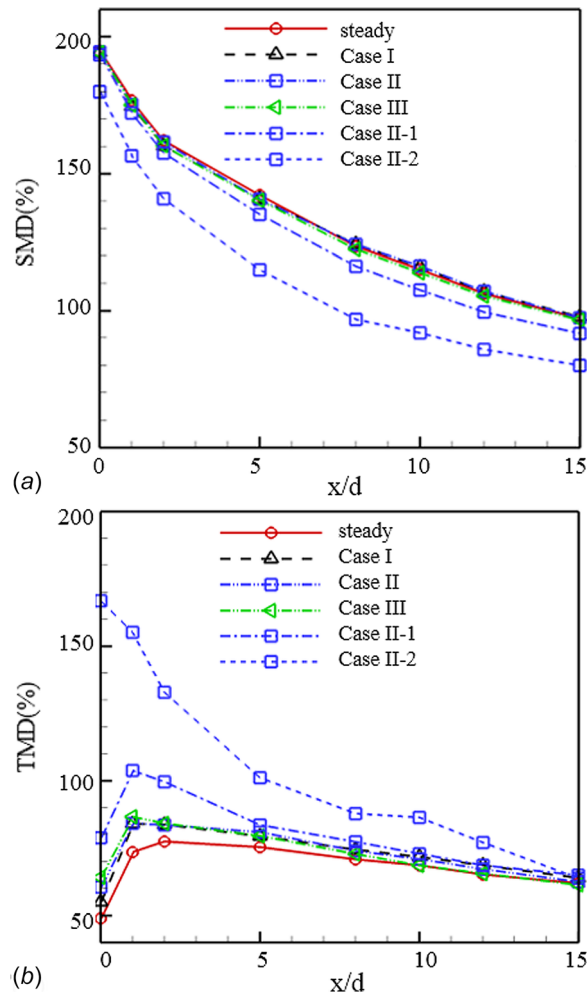


Fig. 11 Spatial evolution of mixing indices: (a) SMD and (b) TMD

are consistent with those in the instantaneous vorticity and scalar fields in Figs. 2 and 3. The oscillating crossflow not only induces bulk shedding of vorticity but also makes the jet plume swing and flap at the forcing frequency. Similar observations are also derived in other forcing cases, at frequencies of 2 and 10 kHz.

The acoustically induced variations are also calibrated by time histories of pressure fluctuations at three locations: close to the rigid jet inlet boundary at $z/d = -39.0$, near the jet exit at $z/d = -2.0$, and in the far field at $x/d = 10.0$. The frequencies and magnitudes of the oscillation are identified by the FFT calculations shown in Fig. 8. The three plots in Fig. 8(a) show the cases with a 10% velocity oscillation at three forcing frequencies, namely, cases I, II, and III, while those in Fig. 8(b) compare the cases with different oscillation amplitudes at the same frequency of 5 kHz in cases II, II-1, and II-2. For all cases, the corresponding forcing frequencies are observed with varying amplitudes at the probe locations. Since perfect wave reflection is applied at the jet pipe inlet at $z/d = -40.0$, a pressure peak is expected, and it is seen in the relatively higher values at $z/d = -39.0$ in the cases with 2 and 5 kHz in the lower left plot. In the case of 10 kHz, on the other hand, the imposed excitation resonates with the intrinsic flow instabilities near the jet orifice and generates a strong fluctuation at $z/d = -2.0$.

3.3 Mean Flow and Scalar Fields. Figure 9 shows time-averaged trajectories of the jet in the jet-center plane based on the streamlines passing through the center of the injection orifice and

the plume boundaries marked by a scalar concentration of 0.05. Results from all five unsteady cases and the baseline steady case are overlaid here for direct comparison. The three low amplitude forcing cases have the same jet trajectory and the same plume size as those in the steady crossflow case. This observation is consistent with the results of the experimental studies in Refs. [8] and [16]. Experimentally, for $r \leq 4$, it is found that the transverse jet may already be self-excited, and the imposition of weak-to-moderate sinusoidal excitations has little effect on jet penetration or spread. Imposition of strong forcing, especially with a distinct, externally imposed time scale such as that created by a square-wave excitation with a prescribed temporal pulse width, is required to impact jet penetration and excitation [6]. Compared to the steady case, the trajectory in case II-2 at a strong oscillation deviates significantly in the immediate downstream of the jet-core zone, showing a center streamline about $1.0d$ lower. The jet plume boundary shows lower bottom and higher upper edges, which suggests that the jet plume is widened by the effect of external excitation.

Figure 10 shows the time-averaged scalar distribution in the jet-center plane and transverse planes at $x/d = 2, 5$, and 10 . The intrinsically asymmetric motion of the CVP accounts for the spanwise asymmetry. Case II shows little difference from the steady case; in case II-2, however, the jet core shrinks noticeably and the plume is expanded and elongated in the transverse direction. The flapping mode identified in the POD analyses enhances the transverse jet dispersion and suppresses the spanwise development, as suggested by the long, narrow mixing zone relatively close to the bottom edge of the domain [9].

Figure 11 shows the spatial mixing deficiency (SMD) and the temporal mixing deficiency (TMD) calculated at several downstream locations for all cases. It is observed that variation in the excitation frequency does not affect SMD, and all three forcing cases at 10% velocity oscillation remain comparable to the steady case. Although TMD (slightly) increases with excitation frequency in all the forcing cases, cases I, II, and III almost converge, and no significant frequency dependency is observed. On the other hand, both SMD and TMD change noticeably as the excitation magnitude increases from 10% to 50%. With a stronger excitation, the jet plume widens, inducing a lower SMD, and the flapping and detaching motions strengthen, yielding a higher TMD. In a nutshell, as the excitation magnitude increases, the jet flow field becomes more spatially homogeneous and temporally heterogeneous. Note that the values of mixing indices are sensitive to the focus zone, and the calculations presented here are confined to the regions $-4 \leq y/d \leq 4$ and $0 \leq z/d \leq 11$.

4 Conclusions

The flow dynamics and intrinsic instabilities in a turbulent gaseous jet in an air crossflow are numerically investigated at a jet-to-crossflow velocity ratio of 4. Cases with both steady and oscillating crossflows are analyzed.

For the case with a steady crossflow, the fluctuating pressure and scalar fields are examined to extract the intrinsic instabilities. Proper orthogonal decomposition (POD) analyses are used to identify the most dynamic modes. It is found that the top two modes are closely linked to the shear-layer vortices near the injector orifice and the vertical movements in the jet wake region, respectively. Their dominant frequencies, 92 and 13.5 kHz, are consistent with the results from the power spectral density (PSD) analyses.

For cases with oscillating crossflows, sinusoidal velocity oscillations are imposed on the crossflow upstream at a variety of forcing frequencies and amplitudes. Results on flow dynamics and scalar mixing show that

- (1) low amplitude external excitations have little impact on jet penetration and evolution for $r = 4$.
- (2) moderate and high amplitude excitations affect the behaviors of the jet plume in the crossflow. The strong vorticity

generation and subsequent breakdown shorten the jet core and reduce the plume contiguity. The POD analyses reveal that the flapping–detaching motions at the forcing frequency and subharmonics play dominant roles in the jet evolution, yielding a low jet trajectory and broader plume.

- (3) externally imposed velocity oscillations in the crossflow influence the development of the counter-rotating vortex pair. The enhanced transverse motions and the suppressed spanwise motions lead to a long, narrow jet plume in any transverse plane, and the mixing zone is relatively close to the bottom edge of the domain.
- (4) both the spatial mixing deficiency (SMD) and the temporal mixing deficiency (TMD) show uniformity among cases with different forcing frequencies at the same low amplitude. At the same forcing frequency, as the excitation amplitude increases, SMD decreases and TMD increases, as a result of enlarged plume size and reduced contiguity.

Acknowledgment

This work was sponsored by the National Energy Technology Laboratory, U.S. Department of Energy under Grant No. DE-FG26-07NT43069.

Nomenclature

a	= sound speed, m/s
C	= jet fluid mass fraction
d	= jet diameter, mm
f	= temporal frequency, Hz
J	= jet-to-crossflow momentum ratio
k	= wave number, m^{-1}
M	= Mach number
P, p	= pressure, atm
r	= jet-to-crossflow velocity ratio
Re	= Reynolds number
St	= Strouhal number
t	= time, s
T	= temperature, K
U, u, v, w	= velocity components, m/s
x, y, z	= spatial coordinates, mm
α	= relative magnitude
γ	= heat capacity ratio
λ	= wavelength, m
ρ	= density, kg/m^3
ω	= angular frequency, rad/s

Subscripts

a	= acoustic
c	= crossflow
d	= jet diameter
e	= excitation
F	= forcing

j	= jet
0	= mean

In the POD Analysis

a	= temporal variation
f	= flow property
j	= mode number
ϕ	= basis function

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